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A SUBOPTIMUM APPROACH TO ADAPTIVE ARRAY PROCESSING

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ABSTRACT

A system that approximates the performance of the optimum processor (optimum in the sense of maximizing array gain) is presented. Consisting of many two-element subsystems, the system is called a binary array processor (BAP). The two main advantages of the BAP system are that: (1) It should be easier to implement than the optimum processor because inversion of large matrices is not required; and (2) It should not suffer from errors made in the statistical estimation of cross-spectral densities of the noise field because the system adapts against such errors. These advantages are obtained at the expense of a few db of array gain, but a sample calculation illustrates that the array gain of the BAP system can still be significantly greater than that obtained by conventional time-shift-and-sum beam formation.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

A SUBOPTIMUM APPROACH TO ADAPTIVE ARRAY PROCESSING

INTRODUCTION

The application of optimum space-time processing theory requires the inversion of a matrix containing N^2 elements at each frequency, where N is the number of hydrophones in the array. Such matrix inversion introduces two serious problems: (1) The implementation of an optimum system would involve an impractical amount of hardware; and (2) Since the largest gains from the optimum processor are obtained when the matrix approaches singularity, any errors in the statistical estimates of the matrix elements could have a drastic effect on the inverse matrix. Even the inverse of an extremely well-conditioned matrix could be affected severely by estimation errors because of the large number of multiply/add operations involved in its calculation.

This report presents a suboptimum scheme that should not suffer from measurement error because the system adapts against estimation errors it has made. Furthermore, the largest matrix the system must invert is a 2×2 matrix (such inversion is trivial), and hence the system should be much easier to implement than the optimum processor. The only constraint on the suboptimum system is that the number of hydrophones in the array must be some power of 2.

DISCUSSION

We first assume that the array has been electronically steered so that the desired signal is identical at each input. In the straightforward approach to optimum array processing, a linear filter is placed in each channel (Fig. 1), and then the question is asked, "What must the transfer functions of these filters be in order to maximize the array gain?" The required mathematical

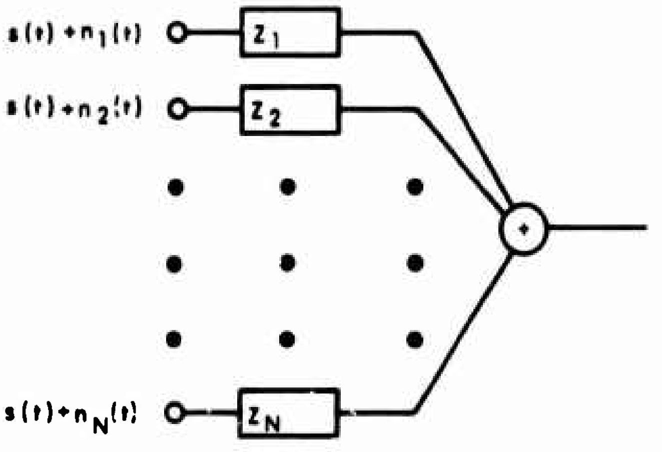

Fig. 1 - Configuration of Optimum Processor

calculations then yield the optimum complex weights[1] at each frequency:

$$Z = Q^{-1}\,\alpha\,, \tag{1}$$

where

Z is an $N \times 1$ column vector of the optimum weights,

Q is an $N \times N$ matrix of the cross-spectral densities between inputs (after steering), and

$$\alpha = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$

A system that approximates the performance of the optimum processor is shown in Fig. 2. Consisting of many two-element subsystems, the system is called a binary array processor (BAP). The BAP is constrained in that the number of hydrophones must be some power of 2, but for many applications this constraint is not a serious one.

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[1]D. J. Edelblute, J. M. Fisk, and G. L. Kinnison, "Criteria for Optimum-Signal-Detection Theory for Arrays," Journal of the Acoustical Society of America, vol. 41, no. 1, January 1967, pp. 199-205.

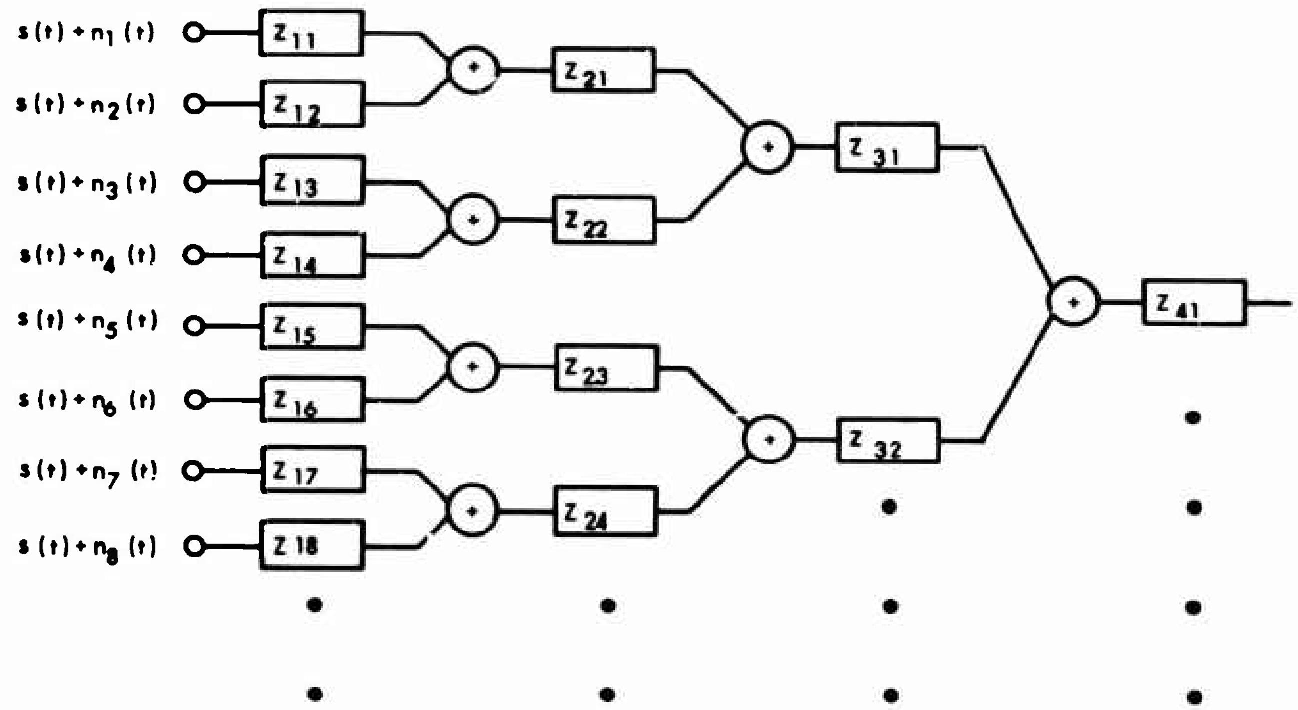

Fig. 2 - Configuration of Binary Array Processor

The BAP system operates in the following manner: First the subset of filters $\{Z_{1i}\}$, where $i = 1, 2, \cdots, 2^M$ and $M = \log_2 N$, is determined by treating each two-element subsystem separately, according to Eq. (1). After these filters have been determined and their outputs have reached a steady-state condition, the subset $\{Z_{2i}\}$, $i = 1, 2, \cdots, 2^{(M-1)}$, is determined in the same fashion. The procedure is repeated for the subset $\{Z_{3i}\}$, $i = 1, 2, \cdots, 2^{(M-2)}$, and so on until all the filters have been determined. An important feature of the BAP system is now evident. Since the filters $\{Z_{2i}\}$ are designed after the filters $\{Z_{1i}\}$, they adapt against errors in the latter set caused by incorrect statistical estimates of cross-spectral densities. Similarly, the filters $\{Z_{3i}\}$ adapt against errors made in determining the filters $\{Z_{2i}\}$, and so on.

Another important feature of the BAP system is that it never requires inversion of a large matrix; many 2×2 matrices must be inverted, but the operations involved are trivial. Hence the BAP system should be orders of magnitude simpler to implement than the optimum system.

ANALYSIS OF THE BAP SYSTEM

Although it is possible to analyze the BAP system for the general case, we can simplify the analysis greatly by assuming that the input-noise cross-spectral density matrix, in addition to being Hermitian, is a Toeplitz matrix, that is, it has equal elements along any diagonal. Only this special case, for which the transfer functions of the filters in each binary subsystem are complex conjugates, is examined in this report. Many filters have identical transfer functions, as given by the following relation:

$$Z_{ki} = Z_{k(i+2)}, \qquad k = 1, 2, \cdots, [M-1]$$
$$i = 1, 2, \cdots, [2^{(M-k+1)} - 2] .$$

Figure 3 shows the BAP processor configuration for the special case under consideration.

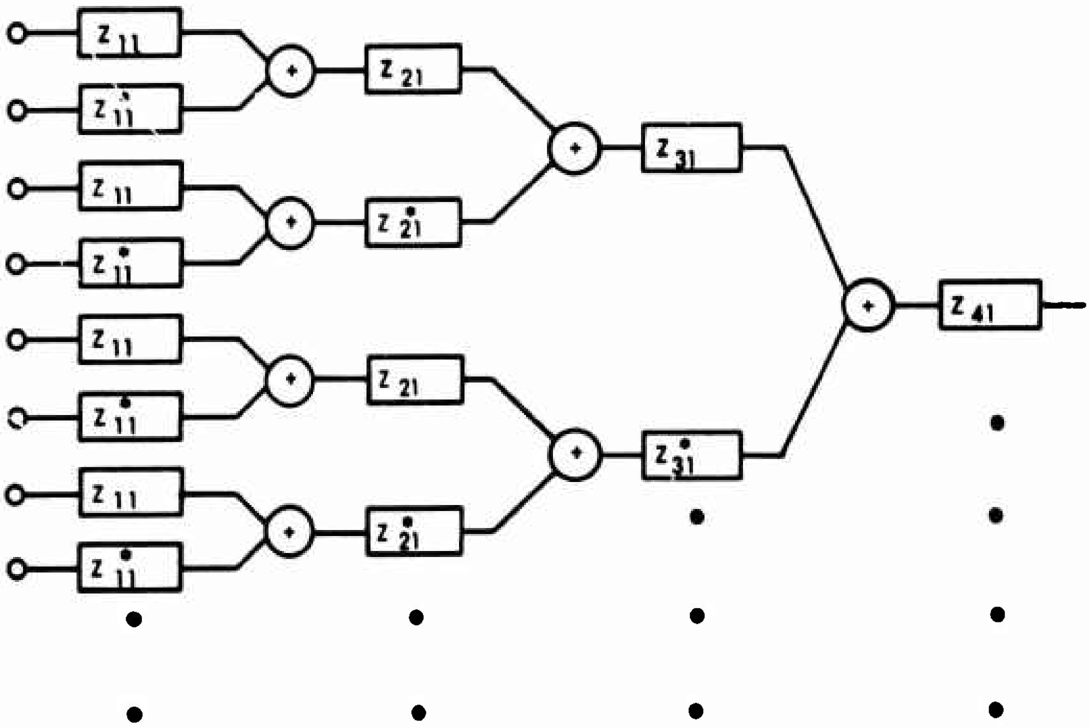

Fig. 3 - Binary Array Processor for Case when Input Cross-Spectral Density Matrix Is Toeplitz

Before proceeding further with the analysis, we must present the notation that will be used for all cross-spectral density matrices. At the input, i.e., just before the filters $\{Z_{1i}\}$, we have:

$$QA = \begin{bmatrix} QA_{11} & \cdots & QA_{1(2M)} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ QA_{(2M)1} & \cdots & QA_{(2M)(2M)} \end{bmatrix}$$

and just before the filters $\{Z_{2i}\}$,

$$QB = \begin{bmatrix} QB_{11} & \cdots & QB_{1(2M-1)} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ QB_{(2M-1)1} & \cdots & QB_{(2M-1)(2M-1)} \end{bmatrix}$$

The matrix of spectra preceding the filters $\{Z_{3i}\}$ is QC, and so on.

Analysis of the BAP system proceeds as follows. Given QA, the filters $\{Z_{1i}\}$ are determined, and then the matrix QB is found; similarly the filters $\{Z_{2i}\}$ and the matrix QC are determined. The process continues until the power spectrum of the final output stage is known. The array gain can then be calculated after this same procedure is repeated using the same filters with signal (instead of noise) cross-spectral density matrices.

When QA is a Hermitian, Toeplitz matrix, the filters $\{Z_{1i}\}$ are all determined once Z_{11} is known, and Z_{11} is found by solving the following matrix equation:

$$\begin{bmatrix} Z_{11} \\ \\ Z^{*}_{11} \end{bmatrix} = \begin{bmatrix} QA_{11} & QA_{12} \\ \\ QA^{*}_{12} & QA_{11} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \\ 1 \end{bmatrix}$$

Now since QB is also a Hermitian, Toeplitz matrix, it is necessary only to find QB_{11}, QB_{12}, QB_{13}, $\cdots$, $QB_{1(2M-1)}$ in order to determine QB. The element QB_{11} is given by the following expression from linear system theory[2]:

$$QB_{11} = QA_{11}\,|Z_{11}|^2 + QA_{12}Z^{*2}_{11} + QA_{21}Z^2_{11} + QA_{22}\,|Z^{*}_{11}|^2$$

[2] J. Bendat and A. Piersol, Measurement and Analysis of Random Data, John Wiley and Sons, N. Y., 1966, Eq. 3.166, p. 108.

With the use of the relations

$$QA_{11} = QA_{22}$$

and

$$QA_{21} = QA^{*}_{12}$$

we reduce this equation to

$$QB_{11} = 2QA_{11}\left|Z_{11}\right|^{2} + 2\,\mathrm{Re}\left[QA_{12}Z^{*2}_{11}\right] \qquad (2)$$

The element QB_{12} is derived here because no expression is readily available in the literature. In Fig. 4 the filters of the first four channels are

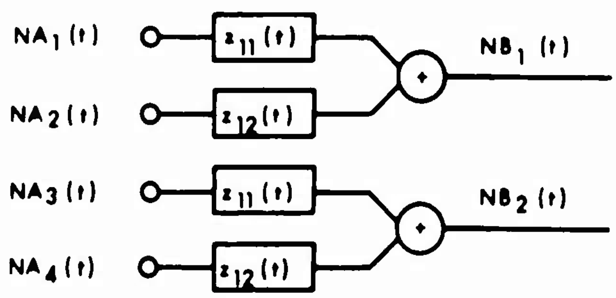

Fig. 4 - Derivation of QB_{12}

represented by their impulse responses instead of their transfer functions. The functions $NA_1(t)$, $NA_2(t)$, $NA_3(t)$, $NA_4(t)$, $NB_1(t)$, and $NB_2(t)$ are noise time functions. The expressions for $NB_1(t)$ and $NB_2(t)$ are

$$NB_1(t) = \int_{-\infty}^{\infty} \left[z_{11}(\sigma)NA_1(t-\sigma) + z_{12}(\sigma)NA_2(t-\sigma)\right] d\sigma$$

and

$$NB_2(t) = \int_{-\infty}^{\infty} \left[z_{11}(\rho)NA_3(t-\rho) + z_{12}(\rho)NA_4(t-\rho)\right] d\rho\ ,$$

and the cross-correlation function between these two functions is

$$RB_{12}(\tau) = \overline{NB_1(t)NB_2(t+\tau)} \ ,$$

where the bar denotes statistical averaging. Substitution of the expressions for $NB_1(t)$ and $NB_2(t)$ into this equation gives

$$RB_{12}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{[z_{11}(\sigma)NA_1(t-\sigma) + z_{12}(\sigma)NA_2(t-\sigma)][z_{11}(\rho)NA_3(t-\rho+\tau) + z_{12}(\rho)NA_4(t-\rho+\tau)]}\, d\sigma d\rho$$

$$RB_{12}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [z_{11}(\sigma)z_{11}(\rho)RA_{13}(\tau+\sigma-\rho) + z_{12}(\sigma)z_{11}(\rho)RA_{23}(\tau+\sigma-\rho)$$
$$+ z_{11}(\sigma)z_{12}(\rho)R_{14}(\tau+\sigma-\rho) + z_{12}(\sigma)z_{12}(\rho)RA_{24}(\tau+\sigma-\rho)]\, d\sigma d\rho \ ,$$

where

$$RA_{13}(\tau+\sigma-\rho) = \overline{NA_1(t-\sigma)NA_3(t-\rho+\tau)} \ ,$$

$$RA_{23}(\tau+\sigma-\rho) = \overline{NA_2(t-\sigma)NA_3(t-\rho+\tau)} \ ,$$

$$RA_{14}(\tau+\sigma-\rho) = \overline{NA_1(t-\sigma)NA_4(t-\rho+\tau)} \ , \quad \text{and}$$

$$RA_{24}(\tau+\sigma-\rho) = \overline{NA_2(t-\sigma)NA_4(t-\rho+\tau)} \ .$$

The Fourier transform of $RB_{12}(\tau)$ is the quantity we desire:

$$QB_{12} = \int_{-\infty}^{\infty} RB_{12}(\tau)\, e^{-i2\pi f\tau}\, d\tau$$

Substituting the previously given expression for $RB_{12}(\tau)$ into the above integral and multiplying and dividing the integrand by $e^{-i2\pi f(\sigma-\rho)}$ yields

$$QB_{12} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_{11}(\sigma)\, e^{j2\pi f\sigma}\, z_{11}(\rho)\, e^{-j2\pi f\rho}\, RA_{13}(\tau+\sigma-\rho)\, e^{-j2\pi f(\tau+\sigma-\rho)}\, d\sigma d\rho d\tau$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_{12}(\sigma)\, e^{j2\pi f\sigma}\, z_{11}(\rho)\, e^{-j2\pi f\rho}\, RA_{23}(\tau+\sigma-\rho)\, e^{-j2\pi f(\tau+\sigma-\rho)}\, d\sigma d\rho d\tau$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_{11}(\sigma)\, e^{j2\pi f\sigma}\, z_{12}(\rho)\, e^{-j2\pi f\rho}\, RA_{14}(\tau+\sigma-\rho)\, e^{-j2\pi f(\tau+\sigma-\rho)}\, d\sigma d\rho d\tau$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} z_{12}(\sigma)\, e^{j2\pi f\sigma}\, z_{12}(\rho)\, e^{-j2\pi f\rho}\, RA_{24}(\tau+\sigma-\rho)\, e^{-j2\pi f(\tau+\sigma-\rho)}\, d\sigma d\rho d\tau\,,$$

which reduces to

$$QB_{12} = Z^{*}_{11}(f) Z_{11}(f) QA_{13} + Z^{*}_{12}(f) Z_{11}(f) QA_{23}$$
$$+ Z^{*}_{11}(f) Z_{12}(f) QA_{14} + Z^{*}_{12}(f) Z_{12}(f) QA_{24}\,.$$

Now since

$$Z_{12}(f) = Z^{*}_{11}(f),$$

$$QA_{23} = QA_{12}\,,$$

and

$$QA_{24} = QA_{13}\,,$$

the element QB_{12} becomes

$$QB_{12} = 2\,QA_{13}\,|Z_{11}|^2 + QA_{12} Z_{11}^2 + QA_{14} Z_{11}^{*2}\,.$$

The expressions for QB_{13}, QB_{14}, etc. are obtained from the following general expression, which was derived by changing subscripts in the preceding derivation:

$$QB_{1K} = 2QA_{1(2K-1)} |Z_{11}|^2 + QA_{1(2K-2)} Z_{11}^2 + QA_{1(2K)} Z_{11}^{*2}, \quad k > 1. \tag{3}$$

Thus all the elements of the matrix QB can be determined. In fact, we can now determine the entire BAP system simply by changing subscripts and variable names. For example, the filters $\{Z_{2i}\}$ are found in the same manner as the filters $\{Z_{1i}\}$, the only difference being that elements of QB are used instead of elements of QA. Similarly, the elements of QC are obtained from the same expressions used for QB simply by changing all the A's to B's and all the B's to C's. These procedures are repeated until all the filters are determined and the value of the output-noise power spectral density is found. Then the same filters and equations with noise cross-spectra replaced by signal cross-spectra are used to find the value of the output-signal power spectral density. Array gain is then computed by dividing the ratio of signal power spectral density to noise power spectral density at the output by the same ratio at an input reference hydrophone.

SAMPLE CALCULATION

Consider a linear array oriented vertically in a surface-generated noise field and having four elements. The following expression,[3] which has been normalized with respect to a selected input reference quantity QA_{11}, defines the elements of the matrix QA :

$$QA_{mn} = 2\left[\frac{\sin \beta_{mn}}{\beta_{mn}} + \frac{\cos \beta_{mn} - 1}{(\beta_{mn})^2}\right] - j\,2\left[\frac{\sin \beta_{mn}}{(\beta_{mn})^2} - \frac{\cos \beta_{mn}}{\beta_{mn}}\right], \quad m \leq n .$$

In this expression,

$$\beta_{mn} = 2\pi\left(\frac{d_{mn}}{\lambda}\right),$$

d_{mn} is the distance between the m*th* and n*th* receivers in feet, and

λ is the wavelength in feet .

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[3]B. F. Cron and C. H. Sherman, "Addendum: Spatial-Correlation Functions for Various Noise Models," Journal of the Acoustical Society of America, vol. 38, no. 5, November 1965, p. 885.

Since we want QA to be Toeplitz, let d_{mn}/λ be constant for all (m, n). For $d_{mn}/\lambda = 0.3$, the elements that specify the matrix QA are:

$$QA_{11} = 1.0 ,$$

$$QA_{12} = 0.272 - j(0.864) = 0.906 \underline{/-72.5^o} ,$$

$$QA_{13} = -0.568 - j(0.347) = 0.667 \underline{/211.4^o} ,$$

and

$$QA_{14} = -0.22 + j(0.323) = 0.391 \underline{/124.2^o} .$$

The filters $\{Z_{1i}\}$ can all be found from Z_{11}, which is given by

$$Z_{11} = QA_{11} - QA_{12} = 0.728 + j(0.864) = 1.13 \underline{/49.8^o} .$$

The matrix QB is determined once QB_{11} and QB_{12} are known, and they are found from Eqs. (2) and (3):

$$QB_{11} = 2QA_{11} |Z_{11}|^2 + 2\mathrm{Re}\,[QA_{12} Z_{11}^{*2}] = 0.26 ,$$

$$QB_{12} = 2QA_{13} |Z_{11}|^2 + QA_{12} Z_{11}^2 + QA_{14} Z_{11}^{*2} ,$$

and

$$QB_{12} = 0.025 - j(0.155) = 0.155 \underline{/-80.85^o} .$$

The filters $\{Z_{2i}\}$ are all found from Z_{21}, which is given by

$$Z_{21} = QB_{11} - QB_{12} = 0.235 + j(0.155) = 0.282 \underline{/33.4^o} ,$$

and the noise-output power spectrum is found from Eq. (2) to be

$$QC_{11} = 2QB_{11} |Z_{21}|^2 + 2\mathrm{Re}\,[QB_{12} Z_{21}^{*2}] = 0.205 .$$

The above procedure is now repeated with the same filters but with signal cross-spectra in place of the noise cross-spectra. We have assumed that the signal is identical at each input; therefore, the input-signal cross-spectral density matrix, normalized with respect to SA_{11}, is

$$SA = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

The elements SB_{11} and SB_{12} are found from Eqs. (2) and (3) with SA_{11}, SA_{12}, SA_{13}, and SA_{14} replacing QA_{11}, QA_{12}, QA_{13}, and QA_{14}, respectively. They are

$$SB_{11} = 2SA_{11}\,|Z_{11}|^2 + 2\mathrm{Re}\,[SA_{12}Z_{11}^{*2}] = 2.143$$

and

$$SB_{12} = 2SA_{13}\,|Z_{11}|^2 + SA_{12}Z_{11}^2 + SA_{14}Z_{11}^{*2} = 2.143\ .$$

Thus the matrix SB is determined. Equation (2) with the appropriate substitutions then yields the signal-output power spectrum as:

$$SC_{11} = 2SB_{11}\,|Z_{21}|^2 + 2\,\mathrm{Re}\,[SB_{12}Z_{21}^{*2}] = 4.74\ .$$

The array gain of the system is, therefore,

$$G = \frac{SC_{11}}{Q'_{11}} = 23.1 = 13.6\text{ db}\ .$$

The above sample calculation has showed an array gain of 13.6 db for a four-element BAP system operating with a vertical array in a surface-generated noise field. Optimum processing under the same conditions would give an array gain of 19 db, whereas conventional time-shift-and-sum beam formation would give less then $10\log_{10}(4) = 6$ db. Thus, although the BAP system may not result in as large an array gain as the optimum processor, it still provides significantly higher gains than are obtainable by conventional methods.

CONCLUSIONS

A suboptimum approach to array processing has been described. The system, called a "binary array processor" (BAP), consists of a collection of many two-element subsystems The relative ease of BAP implementation (resulting from the fact that inversion of large matrices is not required) should compensate for the sacrifice of array gain. In addition, the BAP system should be less sensitive than the optimum processor to estimation errors in the noise cross-spectral densities because it adapts against such errors.

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